Social Science Research 85 (2020) 102367

Contents lists available at ScienceDirect

Social Science Research

journal homepage: http://www.elsevier.com/locate/ssresearch

Nonlinear models of distribution of talking in small groups

Barbara F. Meeker

Department of Sociology, University of Maryland, College Park, MD, 20742, USA

ARTICLE INFO

Keywords:
Group processes
Discussion groups
Nonlinear dynamical systems
Status generalization
Contributive fairness

ABSTRACT

This paper develops a mathematical model of the distribution over time of talking in discussion groups. Researchers of small group processes and social inequality have long recognized that interaction in small discussion groups is usually not equally distributed and that being a person who talks more than others is associated with having higher status outside the group and greater prestige and influence within the group. There is also a history of mathematical approaches to describing this phenomenon. As an addition to this literature, here a nonlinear dynamical system model is presented and used to develop computer simulations that are compared with data from a laboratory study of real four-person discussion groups. The model is based on theoretical assumptions about group processes including individual differences in volubility, status generalization, deference hierarchies and norms of taking turns and of fairness. While none of these alone make predictions that match the data, when they are all combined simulations are produced that closely match the data in both changes over time and differentiation among members. The dynamical system using the parameters as estimated for these data reaches a fixed point, which may help understand how groups structures become stable under some conditions but not others.

1. Introduction

In this paper it is shown that data reporting amount of talking by individuals in task oriented discussion groups can be described by equations from a mathematical model of a dynamical system, which in turn are based on theories about the underlying processes governing how much individuals talk in such groups. This requires a combination of theory, formal model, and data, with particular attention to how the assumptions of the model reflect assumptions of the theory and measurement of data. Results can help understand how several underlying processes operate at once to produce and maintain inequality in amount of talking in discussion groups.

Task oriented discussion groups are important, because they are ubiquitous in social life and because many important tasks are carried out by discussion groups (e.g., committees, juries, work teams, elected councils, families). Numerous researchers have over decades studied the distribution of talking in groups, beginning in the mid-20th century with work by Bales and associates (Bales, 1950, 1952; Bales et al., 1951). Amount of talking has been measured in various ways, including: number of complete sentences or statements per speaker, number of seconds or minutes each person talks, number of changes of speaker, number of interruptions. No matter what the measurement, two findings have reliably occurred; first, that the distribution of talking is unequal, and second, that in general a group member who talks more than others has more influence on the group's decisions or task outcomes. Furthermore, it has also been documented that talking more than other group members is associated with having higher social status outside the group as well as within it (Berger et al., 1972, 1986). Thus, understanding the processes that produce different amounts of talking is important in understanding emergence and maintenance of social inequality at both micro and macro levels.

E-mail address: bfmeeker@umd.edu.

https://doi.org/10.1016/j.ssresearch.2019.102367
Received 9 April 2019; Received in revised form 7 September 2019; Accepted 7 October 2019
Available online 11 October 2019

The current paper adds to the mathematical tools available for the analysis of group interaction by presenting and testing a nonlinear dynamical system model for distribution of group interaction over time. Section 2 below reviews theoretical research about sources of inequality in group interaction; in Section 3, these processes are translated into four models each described by a set of equations that predict the amount of talking by individual group members in successive minutes of group discussion; section 4 presents data from a laboratory study of four-person discussion groups; in Section 5, the equations of Section 3 are translated into a set of computer simulations using parameters estimated from the first several minutes of the data described in Section 4. The simulations show that a model that combines several processes is best for replicating both changes in amount of talking over time and differentiation among group members. In Section 6 several thought experiments show how the model can be used to extend theory about variations in group structure and process.

2. Theoretical background: why do some people talk more than others in a group discussion?

2.1. Individual differences

Why does one person talk more than others in an otherwise unstructured group? The usual layperson's response to this question is "some people are just more talkative than others". In fact, there is evidence that there exists a stable trait of individuals, "response latency" or "dominance", which is the speed with which a person begins to speak when an opportunity occurs (e.g., Conner, 1977; Berger et al., 1986; Fisek et al., 2005). However, the actual rate of talking in any social setting is also influenced by the setting; in the right circumstances a very talkative person may be very silent or vice versa. Evidence exists that the rate of talking of an individual persists when the individual is moved from one group to another, but there is also evidence that rate of talking of one individual is affected by the talkativeness of other group members (e.g., Borgatta and Bales, 1953). The models developed here make the assumption that each group member begins with an inherent trait related to talkativeness, but this trait combines with other factors to produce the actual number of acts by group members. To avoid confusion, an inherent and stable personality trait associated with higher or lower rates of talking will be referred to as "volubility" and the actual rate of talking in a specific group setting as "talkativeness". Thus, the "most talkative" group member is the one who is observed to talk more than others while the "most voluble" is the one with the greatest underlying (unobservable) tendency to talk.

2.2. Expectation states and status generalization

Relative rate of talking in a group is both an independent variable (leading to influence and prestige within the group) and a dependent variable (affected by external status differences). To explain this it is useful to turn to research programs on expectation states and status generalization (some of the important references here are: Berger et al., 1974, 1972; 1986; Fisek et al., 2005; Ridgeway, 2014; Skvoretz and Fararo, 1996, 2016; Skvoretz et al., 1999). "Expectation States" are the expectations group members have for how much each person's contributions will help the group achieve a goal, relative to the contributions of others. This theory has been developed to apply under scope conditions of group task motivation and collective orientation. That is, if all group members are oriented to working on a task which can have a better or worse outcome, and if everyone benefits from a better outcome, it is to the benefit of all members to encourage and accept contributions of those seen as most competent. Thus, members seen by self and others as more competent will be encouraged to contribute more than others.

Expectations for superior task performance may be due to previous demonstrated competence on the particular task (problem is a broken down car, one individual is known to have fixed this problem before) or external certification in this kind of task (one individual is known to work as an auto mechanic). Also, in the absence of any information about each other, group members may take expressed self-confidence or other features of individual demeanor as an indication of task competence (Fisek et al., 2005). Thus, in a task requiring talking, higher volubility can in the absence of other information lead to higher performance expectations.

Status Generalization Theory is an extension of expectation states theory that proposes that when there are external differences in rank on status characteristics such as gender, ethnicity, social class or formal position within an organization this will, unless explicitly contradicted within the group, create a hierarchy of expected competence and hence an underlying rank order that will affect the observed amount of talking. That is, external status differences, even when actually unrelated to task competence, can assign position in a hierarchy through their effect on task performance expectations. Research shows that such group status structures, however begun, are quite stable once established (Ridgeway and Walker, 1955).

2.3. The deference order: hierarchy, taking turns and fairness

A complementary approach to the question of distribution of interaction in small groups involves the concept of deference order. The regularity of patterns of unequal distribution of interaction routinely observed in discussion groups has intrigued mathematically minded researchers for decades and a variety of models have been proposed for an underlying process that might produce such a distribution (a review can be found in Skvoretz and Fararo, 2016). Among the mathematical approaches that have been proposed is the "deference hierarchy" or "deference order" proposed in the first place by Horvath (1965). According to a deference order model, members of a group follow a standard for the order in which they should speak. In this deference order, given a chance for someone to begin talking, the highest rank actor goes first. Group members wait until the person ahead of them in this deference order either stops speaking or declines to speak. In Horvath's model each group is characterized by a single parameter representing the probability that group members will speak or decline to speak when their turn comes. Simulations using the deference order model are able to produce results superior to a baseline model of individual differences only when compared to actual data (Skvoretz and Fararo, 2016).

When Horvath formulated this model, he made no assumptions about how a deference order develops or changes over time. As work in this area has progressed, it has become apparent that the concept of a deference order is compatible with expectation states/ status generalization theory in that the latter provides a theoretical reason for the former. That is, the person for whom the group has the greatest performance expectations (and/or who has the highest external status) is highest in the deference order, speaks first when an opportunity occurs, and hence speaks oftener. However, there remain theoretical questions: why do other group members, especially those at the bottom of the deference order, talk at all? And, why does the highest rank actor ever stop talking? For a deference order to operate there must be occasions on which a change of speaker can occur which means that members of a discussion group agree to take turns.

Taking turns is a type of role behavior, in which there are normative standards that are endorsed by group members when they agree to work on the task presented, i.e., to reach a group solution to a problem. We begin with the assumption that these normative standards include that only one person can talk at once and that every member should say something. Since only one person can talk at once, time is scarce; at any given moment a group member who talks is using up the time available to other group members. However, speaking also uses up the speaker's own time because taking turns means he/she must pause to wait for a response. As a participant who is not speaking decides whether to begin speaking or remain silent, or one who is speaking decides to continue speaking or stop, norms of deference based on performance expectations or status provide guidance.

This is conceptualized here as operating at three time points: first, the current moment, e.g., "I can't talk now because someone else is talking "; next, the immediate past, e.g., "I have just been talking, but I need to stop to let someone else respond"; and third, the overall discussion, "I have been talking a too much during this whole discussion, I should decrease my rate of talking" or "I have not been talking enough, I should increase my rate of talking".

The third time point, the effect of the history of the whole discussion, supposes that in addition to a deference order there is a normative standard related to the relative frequency of talking. This standard for the comparison of overall rates of talking will be referred to here as a "standard of contributive fairness". This concept is closely related to the much more commonly used concept of "distributive justice" but refers to distribution of contributions rather than ratio of contributions to rewards. To justify including this in a model of group interaction, we note that participants in group discussions often have strong feelings about whether a member has talked too much, or not enough and may take measures to ensure "fair" distribution of participation. Discussion groups may adopt formal procedures for allocating talking time such as Roberts's Rules of Order or, as in the U.S. Supreme Court deliberations, the rule that no one speaks a second time until everyone has spoken once. For another example, early research on groups of strangers with no observable external status differences found some groups in which a member who talked more than others wanted them to (an "overactive deviant" e.g., Bales and Slater, 1955) was resented by other group members. However, resentment against a highly talkative member seems not to appear in groups in which there is some external reason for differentiation (e. g., Burke, 1974).

What standard of contributive fairness should be assumed for task oriented discussion groups? In an otherwise unstructured group with collective and task orientations and no external status differences, the outcome is determined by combination of efforts and rewards are distributed equally. Compatible with this is a contributive standard of equality. On the other hand, when external status differences do exist, an unequal standard for contributive justice makes sense; it will seem "fair" for one member of a four person group to talk more than 25% of the time and others less. In its formal structure this is a "balance", or "control" theory. Such theories have been widely used in sociological theory to help explain identity processes, interpersonal attraction, crowd behavior, organizational behavior, and the behavior of political systems (Robinson, 2007). In control models, feedback processes operate to maintain an ideal ("balanced") state in a social system.

3. A set of mathematical models describing number of speech acts by members of a 4-person discussion group

The mathematical models presented here translate these theoretical assumptions into equations that predict how much each member of a four person discussion group will talk. To do this the length of the group's discussion is divided into equal time units; here, this is a minute (however, the unit is arbitrary). In each minute, each member speaks some number of times. How many? First, assume continuity of behavior; the basic predictor of talking in 1 minute is the amount of talking that member did in the previous minute. This amount is modified by feedback from previous minutes based on constraints imposed by scarcity of time, deference, definition of fairness and individual differences in volubility. Four models are here presented. Model 1 is a baseline model using only the assumption of individual differences. Model 2 assumes individual differences and a standard of contributive fairness but assumes no deference order. Model 3 assumes individual differences and a deference order but no standard of contributive fairness; and Model 4 incorporates all the theoretical assumptions, with several variations in possible assumptions about the role of external status as related to fairness and volubility.

Models 3 and 4 are based on work by Leik and Meeker (1995), Meeker and Leik (1997); and Meeker (2002). They introduced models of human interaction based on models familiar in biology, the Lotka/Volterra "species competition" models (Lotka, 1932; Volterra, 1931). These biological models describe growth or decline of species that must share an ecological space with scarce resources. (A possibly better known version of this kind of model is the "predator-prey" model [foxes and rabbits]. The models presented here do not suppose that anyone is eating anyone else. It may even be misleading to think of it as a variation of a "competition" model because the orientation of the group members is not competitive. Discussion group members have a cooperative orientation but must coordinate within a situation of scarce resources, time being the scarce resource.) The Leik-Meeker models dealt only with a situation of two actors, and did not compare results with any real data. The present paper expands the model to four actors, and uses data from a study of real groups to estimate parameters and test predictions.

3.1. Model 1. baseline model: individual differences

The baseline model for amount of talking in a discussion group assumes only that there are stable individual differences in volubility, resulting in differences in observable behavior in the first minute of the discussion that continue thereafter. Each minute of discussion replicates the previous minute. An equation for this is:

$$x_{i,t} = x_{i,t-1} \tag{1.0}$$

where $x_{i,t}$ is the number of acts by person i in minute t and $x_{i,t-1}$ is the number of acts by person i in minute t-1 (the previous minute).

3.2. Model 2. contributive fairness: equal and unequal

The second theoretical model assumes that taking turns operates through a process in which each actor compares his/her proportion of the talking with an ideal rate, the standard of contributive fairness. When an opportunity to speak occurs, an actor will talk more than in the previous minute if his/her accumulated past rate is below the ideal and less if that rate is above.

Applied to talking in discussion groups this requires a pair of parameters:

Φ_i the ideal "fair share" for actor i, an unobservable interior mental state of actor i

and

$a_{i,t}$ which is i's actual share of talking for all the previous minutes up to minute t. The actual share is calculated as follows:

$$a_{i,t} = \sum_{t=1}^{m-1} x_{i,t} \Big/ \sum_{i=1}^{4} \sum_{t=1}^{m-1} x_{i,t} \tag{2.1}$$

Where m-1 is the number of minutes up to the present one and $a_{i,t}$ is actor i's share. This share is the total number of acts produced by person i over all minutes up to the present divided by the total number of acts produced by all actors. The model then compares $a_{i,t,}$ i's actual share, with Φ_i, i's ideal share, and introduces this into the equations as a factor modifying the actor's talkativeness.

$$x_{i,t} = x_{i,t-1} \left(\Phi_i / a_{i,t-1}\right) \tag{2.2}$$

In Equation (2.2), $x_{i,t}$ is the number of speech acts produced by actor i in minute t; $x_{i,t-1}$ is the number of speech acts produced by actor i in minute t-1, Φ_i is the ideal "fair share" for actor i, and $a_{i,t}$ is i's actual share of talking for all the previous minutes. Note that if i's share is greater than ideal, then $a_{i,t-1}, > \Phi_i$, and $\Phi_i/a_{i,t-1} < 1$ so actor i's output is reduced for minute t, while if i's share is less than ideal, $a_{i,t-1} < \Phi_i$, and $\Phi_i/a_{i,t-1} > 1$ so actor i's output is increased for minute t. If $a_{i,t-1} = \Phi_i$ then $\Phi_i/a_{i,t-1} = 1$ and i repeats the number of acts from the previous minute.

As simplifying assumptions, it is assumed that although different actors may have different expectations for their own fair share of contribution that expectation is constant throughout the interaction, and that in assessing fair share each actor has only his/her own share of contributions to worry about. We will also assume to begin with that the expectations add up to 100%. This assumption can be modified in further use of this model (see Section 6.3). Although theoretically the comparison should be between an actor's *perception* of own and others' relative amounts of talking, in this project we have only measurements of the actual of amount of talking so that is used as the basis for a_i.

3.3. Model 3. deference order and scarcity of time

The next model incorporates scarcity of time, deference and individual differences but without a standard of contributive fairness. Here the question of why group members stop talking as well as why they start or continue is addressed by observing that since group discussion requires turn-taking, time is scarce. The model assumes that talking in the *previous* minute reduces time for *self* in *this* minute ("I must pause for a response or to let someone else have the floor"). To represent this add a parameter S, representing the degree to which an actor's talking on 1 minute is *reduced* by his/her own talking the minute before. The model also assumes that talking in *this* minute reduces time for *other*s in *this* minute ("I can't talk now, someone else has the floor"). For this add another parameter R, representing the degree to which an actor's talking is reduced by *others'* talking in the current minute.

Finally, each actor has a different rate of volubility. To represent volubility the model includes a parameter C_i, representing actor i's volubility. This is not affected by behavior of self or others in past or present minutes and is different for each actor. The values of the parameter R, reaction to talking of others, and the parameter S, effects of self's own talking, are assumed here to be the same for all actors. This assumption reflects the fact that all actors are working on the same task. Different tasks may impose different constraints. Discussion is not the only kind of group task that requires taking turns; another example might be a team sport that requires passing a ball from one player to another in order to score. Also, not all group tasks require taking turns and this model applies only to those that do.

The concept of deference enters when only one actor can speak at once. In a given minute, if a chance to talk occurs, the actor highest in the deference order speaks first and therefore has only his/her own amount of talking in the previous minute to compete with. The second in the deference order has his/her own talking in the previous minute plus the amount talked by person number 1 in the present minute, the third has his/her own previous talking plus time used up by numbers 1 and 2, and the fourth faces competition from acts by all three others. In other words, whenever an opportunity occurs in a given minute, each successive rank actor will fill in time left open by the higher rank actors. Thus, the amount of talking by every actor is reduced by the amount of talking in *that* minute by all actors *higher* in rank. A "chance to talk" may include a pause that can be followed by speech by the same actor; those higher in the deference order will more often continue speaking after a pause than those lower as well as beginning to talk oftener.

$$x_{1,t} = x_{1,t-1} + x_{1,t-1}(-S(x_{1,t-1}) + C_1) \tag{3.1}$$

$$x_{2,t} = x_{2,t-1} + x_{2,t-1}(-R(x_{1,t}) - S(x_{2,t-1}) + C_2) \tag{3.2}$$

$$x_{3,t} = x_{3,t-1} + x_{3,t-1}(-R(x_{1,t} + x_{2,t}) - S(x_{3,t-1}) + C_3) \tag{3.3}$$

$$x_{4,t} = x_{4,t-1} + x_{4,t-1}(-R(x_{1,t} + x_{2,t} + x_{3,t}) - S(x_{4,t-1}) + C_4) \tag{3.4}$$

Here $x_{i,t}$ is the number of speech acts produced by actor i in minute t; $x_{i,t-1}$ is the number of speech acts produced by actor i in minute t-1, C_i is actor i's volubility, R is the degree to which any actor's speech output is reduced by the speech output of others in minute t and S is the degree to which any actor's speech output is reduced by that actor's own speech output in minute t-1.

With only one actor, this equation is an example of the logistic curve, familiar to demographers and epidemiologists because it can describe the growth of populations and epidemics. For example, it shows that as a population grows, it uses up food so that its rate of growth slows. With two actors, the pair of equations becomes a variation of the Lotka/Volterra model of species competition. That shows how the growth of each of two species that share an ecological space is limited by need of both for the same food. In the species competition model, population growth is also affected by the reproductive rate of each species independent of availability of food; in the present model this is parallel to volubility, the parameter C.

3.4. Model 4 all models combined

Models 2 and 3 are now combined in Model 4.

$$x_{1,t} = x_{1,t-1}(\Phi_1 / a_{1,t-1}) + x_{1,t-1}(-S(x_{1,t-1}) + C_1) \tag{4.1}$$

$$x_{2,t} = x_{2,t-1}(\Phi_2 / a_{2,t-1}) + x_{2,t-1}(-R(x_{1,t}) - S(x_{2,t-1}) + C_2) \tag{4.2}$$

$$x_{3,t} = x_{3,t-1}(\Phi_3 / a_{3,t-1}) + x_{3,t-1}(-R(x_{1,t} + x_{2,t}) - S(x_{3,t-1}) + C_3) \tag{4.3}$$

$$x_{4,t} = x_{4,t-1}(\Phi_4 / a_{4,t-1}) + x_{4,t-1}(-R(x_{1,t} + x_{2,t} + x_{3,t}) - S(x_{4,t-1}) + C_4) \tag{4.4}$$

As in all the previous models, $x_{i,t}$ is the number of speech acts produced by actor i in minute t; $x_{i,t-1}$ is the number of speech acts produced by actor i in minute t-1. As in Model 3, C_i is actor i's volubility, R is the degree to which any actor's speech output is reduced by the speech output of others in minute t and S is the degree to which any actor's speech output is reduced by that actor's own speech output in minute t-1. As in Model 2, Φi is the ideal "fair share" for actor i, and $a_{i,t}$ is i's actual share of talking for all the previous minutes up to minute t.

3.5. Some comments on these models

These models describe a system, in the sense that outcomes (amounts of talking) at each time point depend on feedback from outcomes at previous time points. The baseline model, Model 1, supposes no feedback processes, while Models 2, 3 and 4 do. Sociologists have long been interested in groups as social systems with feedback processes, especially feedback resulting in equilibrium (once again going back to work of Bales and associates [e.g. Bales, 1950; Bales et al., 1951; see especially Bales, 1952]). Lacking appropriate mathematical models and high speed computers to run simulations, these early researchers were limited in their ability to model feedback processes and their effects. Nonlinear dynamical systems models have promise to help with this type of theoretical endeavor because a dynamical system model shows effects of feedback on a system over time.

To find mathematical background for dynamical systems models and engineering and biological applications, see any of a number of basic textbooks on dynamical systems or chaos e.g. Strogatz (2015). Some pertinent features can be summarized here. The set of equations (3.1)–(3.4) and (4.1)–(4.4), like other dynamical systems, are "sensitive to initial conditions" meaning that with different values of the parameters R, S, C, and Φ there may be dramatically different long-term outcomes. In general, models like these that are based on the logistic curve may predict outcomes ranging from a stable equilibrium to a periodic or a chaotic (unpredictable) set of outcomes depending on the values of the initial parameters. It has long been known that in the Lotka-Volterra model some combinations of values for the parameters R and S result in an outcome in which the two species co-exist while other combinations result in one species winning the competition while the other disappears.

Nonlinear models differ in several ways from the more usual mathematical approaches to small group behavior such as those described in Skvoretz and Fararo (2016). First, most of the other mathematical models of distribution of talking in groups are stochastic; they assume the underlying process is based on a probability, they seek parameters that express a probability that an action or underlying state will occur, and they make predictions that describe a distribution of outcomes. In contrast, a nonlinear dynamical system model is deterministic; once the parameters have been set, every prediction will be the same and the model predicts exact numbers. Second, since different sets of parameters may produce radically different outcomes, it is not guaranteed that there will be a set of parameters that predict outcomes that are a good match for a particular data set. The basic research question here is whether a set of parameters can be found that when used in the equations can predict some real data. Third, to make predictions from a nonlinear system, one cannot solve the set of equations analytically; the usual method of making predictions using such a set of equations is by computer simulation, which is the method used here.

Finally, we should note that in the language of nonlinear dynamical systems models, all of the models used here are discrete-state, discrete-time dynamical system models, which is theoretically appropriate since at any given moment each person is either talking or not talking.

4. Data from some real groups

The aim of this paper is to see whether simulations based on one of these models can match the actual amount of talking in data from a set of real groups. An appropriate data set is one that was collected by a team directed by John Skvoretz, Murray Webster and Joseph Whitmeyer, as part of a research program designed to develop stochastic models of status generalization in small groups (their methods are described in detail in Skvoretz et al., 1999; Skvoretz and Fararo, 2016).

4.1. Participants, setting and task

The data are from four-person groups, composed of university undergraduates at a southern state university, who were recruited through campus advertisements to be in a study for pay. All groups were same-sex, and participants were previously unacquainted. The groups' task was one of a type commonly used in small group experiments, a "survival" problem. In this case the problem was to rank order 15 items (e.g., water, a flashlight, a Geiger counter) according to usefulness in a fallout shelter in the event of nuclear attack. The groups were asked to talk until they reached consensus and were informed that their decisions would be evaluated by their reasoning as well as how nearly their answers resembled expert opinions. In the analysis that follows, the *unit of analysis* is the group; 50 groups are included.

Initial analysis showed that gender was not significantly associated with any other variables; male and female groups are therefore combined in further analysis. The discussions ranged from 27 minutes to 47 minutes (for all 50 groups combined, mean length = 34.2 minutes, with standard deviation = 6.73). Since groups vary in the number of minutes of discussion, the analysis here is limited to the first 27 minutes of discussion for all groups. No correlation was found between total number of minutes of discussion and any of the other variables considered here.

4.2. Dependent variables: talking

The dependent variables are derived from observations of the verbal activity (talking) of the participants. All verbal activity was measured electronically. The experimenters equipped each group member with a voice-activated recorder, which was attached to a computer system that registered when the person began and finished talking, in fractions of a second. This resulted in a file of "acts" each with a time stamp indicating which person produced the act and the fraction of a second from start of the group discussion the act began and ended. An "act" was defined by the researchers as a continuous speech; "continuous" meaning there is no more than a 1.5 s pause between speeches by the same person. Acts less than 0.33 s were removed from the file to eliminate accidental, non-meaningful noises. The acts as counted in the data file include every time an actor talks, whether or not someone else is also talking at that moment. For purposes of the present project, the data file provided by Skvoretz et al. was transformed into a file of minutes by actors, which consists of the number of acts produced by each actor in each minute.

4.3. Measuring and defining an "act"

Frequency of participation in group discussion has been measured in a variety of ways. The underlying concept is an "act" which is a unit of behavior (usually verbal, although some methods have included non-verbal behavior) that can be identified by an observer as being produced by an individual group member. The phenomenon being thus measured has turned out to be quite robust; almost all measurement techniques have shown similar patterns of differences between most talkative and less talkative group members.

The method used by Skvoretz et al. makes possible measuring both the length of an act in up to hundredths of a second as well as whether an act occurs. It does not include information about other properties of acts such as who is being talked to or the content of the act. All acts are included even if more than one person is talking at the moment. (There are some acts that may be performed by one than one person at once, for example laughing, expressions of agreement or cries of distress. Some overlaps may indicate attempts to interrupt or to reserve a future opportunity to talk.)

In assigning an act to a specific minute, each act is included in only 1 minute, which means that a long act may actually occur in 2 minutes while only being counted in one. This occurs seldom, because acts are in general rather short; the longest is 12.34 s while the shortest is 0.33 s, the smallest number of seconds to be included as an act by the coding system. The data presented below in sections 4.5 and 4.6 for statistical comparisons use the mean for either 5 or 10 minutes, which smoothes out unevenness produced by assigning an act that occurs in 2 minutes to only one of those minutes.

For the first 27 minutes of discussion, the average number of acts per minute by group is 9.99. . Of particular interest in this as in other research is the existence of differentiation, i.e. a rank order of frequency of talking. Such a rank order appears in these data; for all 50 groups the average number of acts per minute by rank from most talkative to least talkative is: most talkative, 3.67, second most talkative, 2.77, third most talkative. 2.167 and least talkative, 1.38. Since this coding system reports duration of acts in seconds, we can also ask whether the most talkative group member not only speaks more often, but also produces longer speeches. The answer is, yes; the average number of seconds per act for minutes 1 to 27 is: most talkative member, 6.61 s per act; second most talkative, 5.62 s per act; third most talkative, 1.18 s per act, and least talkative, 1.09 s per act. As in other groups of this type, the less talkative members produce shorter acts as well as less frequent ones.

Although we can observe the duration of acts as well as their frequency, the frequency of acts is the dependent primary dependent variable in this model. There are several reasons for this. First, frequency of talking has been used in past literature dating back a number of decades so we can compare the present results with existing literature. Second, counting frequency of acts meets the assumptions of the mathematical model. The mathematics in the Lotka-Volterra species competition model is a *discrete-time, discrete state* dynamical system model; it assumes that a member of a species is either alive or dead at any moment. Likewise, we assume that in a discussion group an individual is either talking or not talking at any given moment. A model for a continuous process such as predicting amount of time in a state of talking would require a somewhat different mathematical approach. Third, although it is true that the scarce resource in a discussion group is time to talk, the theory generally refers to the taking of turns, especially to beginning or continuing to talk when an opportunity occurs. That is, theoretically time constraints are experienced by group members as a scarcity of opportunities to take a turn at talking. Counting the frequency of the discrete state of either talking or not talking thus meets the assumptions of group process theory as well as of the mathematical model.

4.4. Independent variable: status

The experiment had three conditions of external status operationalized by both class standing (1st year vs. 4th year students) and an additional manipulation that informed subjects in groups with both 1st and 4th year students that the 4th year students had superior academic records. Subjects were assigned to same-sex groups in one of the following status conditions:

Equal, 4 Low (groups composed of four 1st year students) n = 16 groups.

Unequal, 2 High, 2 Low (groups with two 1st year and two 4th year students) n = 17 groups.

Unequal, 1 High, 3 Low (groups with one 4th year and three 1st year students) n = 17 groups.

4.5. Results from real groups: change in talking over time

The analysis begins with a discussion of distribution of amount of talking over the first 27 minutes of discussion for all 50 groups on average. A striking feature of these results is that the number of acts per minute declines from a group total of about 14 at the beginning to between 8 and 9 acts on minute 27, for all conditions and for all rank actors. This is not just an average; an examination of all of the individual group profiles shows that out of 50 groups there were only 2 that did not have fewer acts on minute 27 than on minute 1. If we compare the average number of acts per group per minute for minutes 1 through 5 (where there is a peak) with that average in the last 10 minutes (minutes 18–27, where number of acts is level) we find a significant difference. In addition to the average number of acts per minute per group we present a 95% confidence interval; if two averages are within the same confidence interval, they do not differ at a 5% level of significance. (Presenting confidence intervals will also allow for comparison of these data with values predicted by computer simulations derived from the models in Section 5.2). For minutes 1–5, the mean is 12.17 with CI [11.38–13.01] while for minutes 18 to 27, the number of acts per minute is 9.27 with CI [8.48–10.06]. These means are not within the same confidence level, so we can say that the behavior of the groups is statistically different at the end of the 27 minutes compared to their behavior at the beginning.

The group profiles also show that there are no groups in which any member completely drops out; although there are groups in which there are minutes on which a member says nothing, that member always resumes talking eventually. Thus, two features to will look for in evaluating the success of a simulation of these data are whether the simulation shows a decline in total number of acts and whether all members continue to participate.

Is a decline in talkativeness over time typical of discussion groups? It is hard to tell, because published research on distribution of talking in discussion groups has almost always presented either a single number describing the average for the whole discussion or a picture of some derived statistic over time (e.g., percent of task vs socio-emotional acts, or percent of acts by most active). The amount of talking may have been decreasing, increasing or stable. Discussion of the meaning of change over time in this type of model appears in Section 6.1 below.

4.6. Summary statistics

In addition to looking at the overall shape of the amount of talking by minute, this analysis looks at the values of two summary statistics for each group. Since the data from the first 5 minutes are used to estimate parameters for the model (described below in Section 5.1), the early minutes are not suitable as a test of the model. The statistical comparisons that will test the models' predictions are behavior in the last 10 minutes, minutes 18–27. The two summary statistics are the average number of acts per minute per group and the percent of acts per produced by the overall most talkative member. Percent of acts by the most talkative member is an indicator of the degree of dominance of the discussion by the group's most talkative member. "Actor 1", the most talkative, as identified in the data is always the actor who spoke the most whether or not that was a group member identified by the experimenter as having superior status.

Initial examination of these statistics for the three conditions showed that there was no statistically significant difference among the three status conditions in average number of acts per minute by group. For percent of acts produced by the most talkative, the two Unequal conditions (2High2Low and 1High3Low) did not differ from each other but had higher percentages than the Equal (4 Low) condition. A higher rate of dominance by most talkative for the two Unequal conditions compared to the Equal condition is as would be predicated by status generalization theory; that is, we would expect that when there is an external status difference, there would be greater differentiation in talkativeness within the group. However, an interpretation of this result as meaning that the subjects in the Unequal conditions accepted the external status difference is contradicted by another feature of the two Unequal groups. This is that an examination of which member actually was the most talkative finds that in 12 of the 34 groups in the Unequal conditions the most talkative member was *not* the one (in the 1High3Low condition) or one of the two (in the 2High2Low condition) identified as having higher external status. Since the 1High3Low and 2High2Low conditions are otherwise similar, the Unequal groups are here placed into categories of "Unequal Consistent" (the 22 groups in which the most talkative was one with higher external status) and "Unequal Inconsistent" (12 groups in which the most talkative was not higher in external status). There were two groups (12.5% of groups) in the 2High2Low condition that were Inconsistent while ten groups (55.6%) in the 1High3Low condition were Inconsistent.

The means for number of acts minutes 18 to 27 for the Equal, Unequal Consistent, and Unequal Inconsistent categories are: Equal, Mean = 9.65, 95%C. I. = [8.32–10.98]; Unequal, Talkativeness Consistent with External Status, Mean = 8.78, 95% C.I. = [7.67–9.89]; Unequal, Talkativeness Inconsistent with External Status, Mean = 9.68' 95% C.I. = [7.37–11.98]. Judging by the confidence intervals, the average number of acts in minutes 18–27 does not differ among the three status conditions. Although these are not statistically different it seems that the groups that did accept the status identification were somewhat less talkative.

The means for percent of acts by most talkative in minutes 18–27 are: Equal, Mean = 35%, 95%C. I. = [32%–38%]]; Unequal, Talkativeness Consistent with External Status, Mean = 40%, 95% C.I. = [36%–43%]; Unequal, Talkativeness Inconsistent with External Status, Mean = 38%' 95% C.I. = [33%–43%]. . Comparing the Equal with the two categories of Unequal, we see that the percent of acts by most talkative is statistically greater for the Unequal Consistent than for the Equal groups, but the Unequal Inconsistent, being halfway between, is not statistically different from either. The Unequal Inconsistent group has a wider confidence Interval for both summary statistics, suggesting more variation among these groups in the development of the group structure.

Past research has shown that 4-person groups show the least amount of participation differentiation compared to other size groups up to about 8. Typically, there is an average of about 32% of total talking from the most talkative member for four-person groups with no external status assignment (Reynolds, 1971, p 705; see also Kadane and Lewis, 1969). This is within the confidence interval for the "Equal" groups, which means the results here are similar to results found by earlier research. It also suggests, however, that a model developed to simulate these four-person groups may underestimate sources of differentiation in larger groups. Replication with minute-by-minute data from larger groups would be useful.

4.7. features that a simulation should reproduce

To look like data from these real groups, results of simulations derived from theoretical considerations expressed in Models 1 through 4 (Section 3.1 to 3.4 above) should show:

- decline in group average from about 14 acts per minute at the beginning to around 8 or 9 on the last minute
- continuous participation by all actors i.e., no one drops out
- differentiation-actors have different rates of talking
- effect of status-the percent of acts produced by the most talkative in a simulation of the Equal category should be about 35% in the last 10 minutes; a simulation of the Unequal, Consistent category should show percent of about 40%; and in a simulation of the Unequal, Inconsistent category a percent of about 38%.

5. Computer simulation

In this section, the equations describing the theoretical models are translated into equations suitable for a computer program to predict amount of talking. Seven simulations are generated; the first is based on the baseline (Model 1); the second and third assume a contributive standard only (Model 2, Cases 1 and 2) comparing equal and unequal contributive standards; the fourth assumes a deference hierarchy and individual differences but no contributive standard (Model 3); and fifth through seven, three cases of Model 4, which combine all assumptions.

5.1. Estimating parameters

In order to run simulations based on the equations in Section 3, it is necessary to have values for the parameters R, S, C, Φ, a starting value for each actor on the first minute, and to make some assumptions about deference order and relative volubility. Values for the parameters are estimated from the data described above from the first 5 minutes of discussion. This involved a series of steps. First, the number of acts per minute for each actor before the simulation begins (minute 1) is estimated by using the average of actual number of acts for each rank actor on minute 1 for all 50 groups. Second, for models 3 and 4, the parameters R and S were directly estimated from amount of talking per minute in the first 5 minutes for all 50 groups combined using a regression model. As stated above in Section 3.3, using the same values of R and S for all actors in all groups is theoretically justified by the fact that all were working on the same task. Third, using these values of R and S, the values of C for the 16 Equal groups were estimated by assuming, as justified theoretically (Section 2.3 above) that where no external status difference exists, groups seek a contributive standard of equality. This estimation began with a value for C of 0.1, a value that Leik and Meeker (1995) reported produced a system with stable but differentiated outputs by two actors. Then, using a Bayesian logic, values were identified that predict actual results for the first 5 minutes in the 16 groups in the Equal condition. Fourth, to find an estimate of Φ for Unequal conditions, the values of C as estimated for the Equal condition were used to get estimates for Unequal values of Φ using data from the first 5 minutes of the 34 groups in the Unequal conditions.

Thus estimated, acts on minute 1 by actor# 1 = 5, by actor# 2 = 4, by actor# 3 = 3, and by actor# 4 = 2. In the models that assume a deference order, the actor with rank #1 on the first minute goes first on subsequent minutes, actor# 2 second, actor# 3 third and actor# 4 last. The parameters are: R = 0.05; S = 0.07; The parameter C (individual volubility) is 0.45 for actor# 1, 0.35 for actor# 2, 0.25 for actor# 3, and 0.15 for actor# 4. The parameter Φ, ideal percent of talking by each actor according to the standard of fairness of contributions, for a standard of equality is 25% for all while for a standard of inequality the values of Φ are: actor #1 = 35%, actor #2 = 25%, actor #3 = 22%, and actor #4 = 18%. These parameters are entered into the four equations of each model and a computer program runs a set of iterations resulting in a prediction of number of speech acts by each actor on each minute from 1 to 27. The question for each simulation is whether the equations, using parameters estimated to fit the first 5 minutes of interaction, will predict behavior in the last 10 minutes compared with the summary statistics (group average and percent by most talkative) described in Section 4.6 in addition to meeting the other criteria (decline in number of acts, continued participation by all actors) listed in Section 4.2.

For each simulation in addition to looking at the figure showing the results to see if there is a decline in acts over time, we test the following hypotheses. (1) The simulation shows that no members'' acts have declined to zero (2) For the simulation, the average number of acts in minutes 18 to 27 will be within the 95% Confidence Interval for number of acts in minutes 18 to 27 in one of the three categories of real groups. As shown in Section 4.6, the means for number of acts minutes 18 to 27 for the Equal, Unequal Consistent, and Unequal Inconsistent categories are: Equal, Mean = 9.65, 95%C. I. = [8.32–10.98]; Unequal, Consistent, Mean = 8.78, 95% C. I. = [7.67–9.89]; Unequal, Inconsistent Mean = 9.68, 95% C.I. = [7.37–11.98]. (3) For the simulation the means for percent of acts by most talkative in minutes 18–27 are within the 95% confidence Interval for one of the categories of real groups. As shown in Section

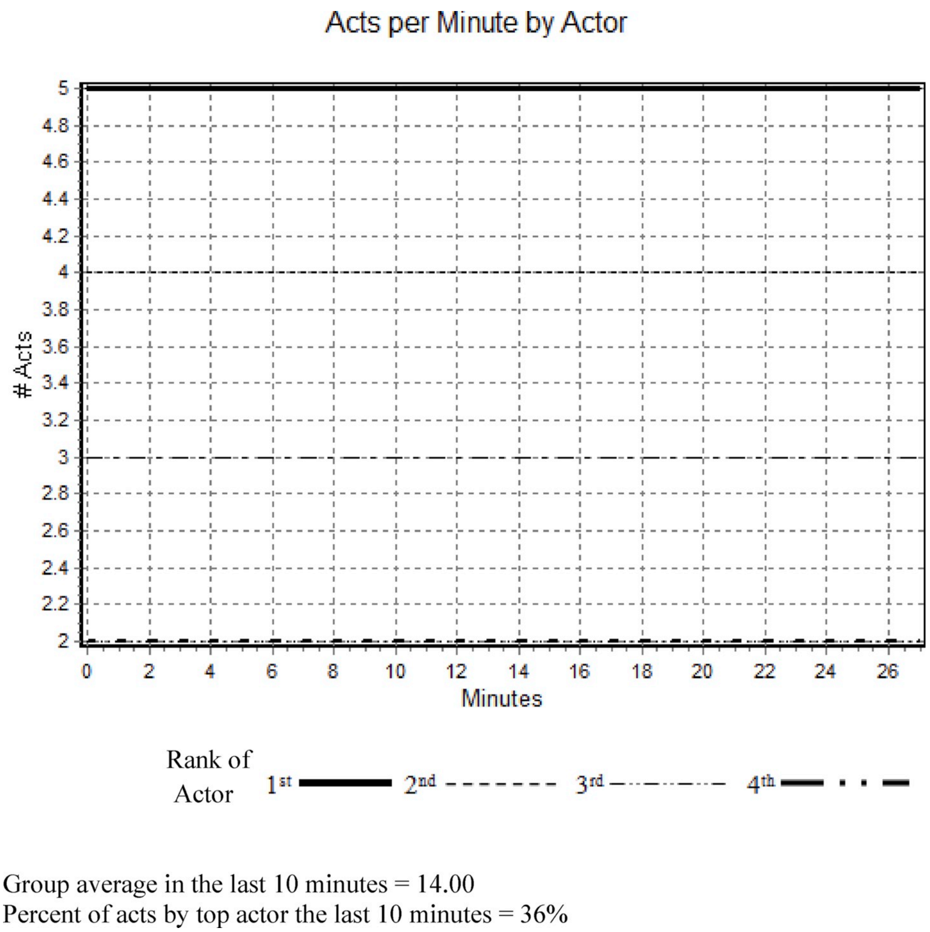


Fig. 1. Model 1. Baseline model, predicted number of acts per minute by actor.

4.6, these are. Equal, Mean = 35%, 95%C. I. = [32%–38%]]; Unequal Consistent, Mean = 40%, 95% C.I. = [36%–43%]; Unequal Inconsistent, Mean = 38%' 95% C.I. = [33%–43%].

As a caveat, we need to remember that these parameters and the predictions derived from them are specific to this data set. A different set of groups, with participants from a different population, with a different task or setting, or a of a different size, might require different parameters and produce different predictions.

5.2. Predictions

5.2.1. Predictions of Model 1, baseline model

The baseline model is based on Equation (1.0) (Section 3.1) with parameters based on the actual average number of acts on minute 1 by actors of each rank; rank is based on behavior in minute 1.

Model 1, Baseline

$$x_{1,t} = 5 \quad (5.1)$$

$$x_{2,t} = 4 \quad (5.2)$$

$$x_{3,t} = 3 \quad (5.3)$$

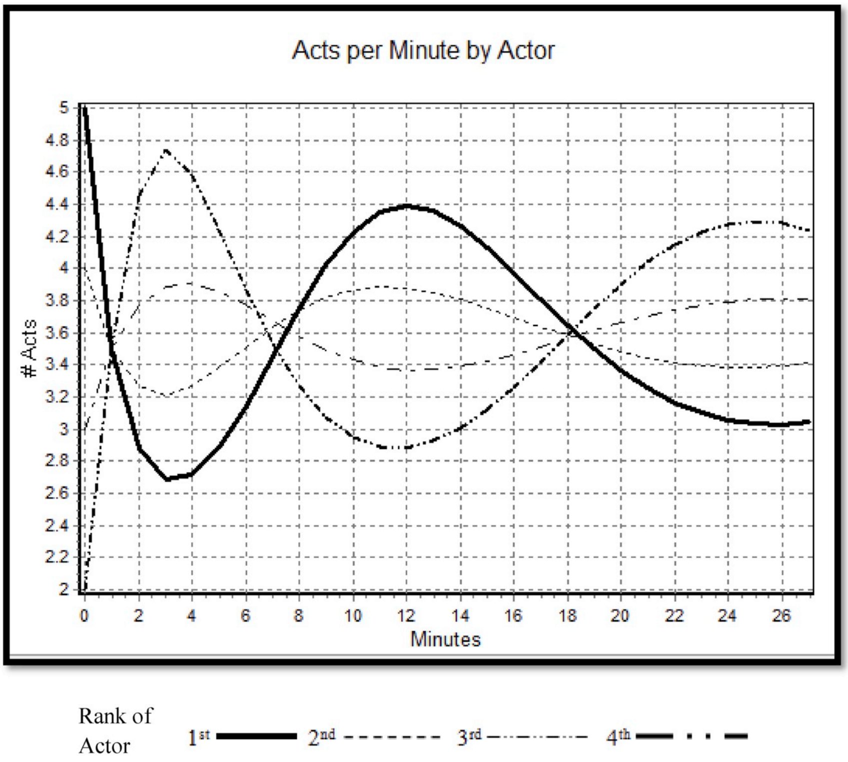
$$x_{4,t} = 2 \quad (5.4)$$

with results shown in Fig. 1.

For the baseline model, the predicted shape in Fig. 1 is flat, with the predicted group total on the last minute being the same as on the first minute, 14.0. The predicted percent by top actor is 36% throughout. Although the baseline model does predict differentiation and continuous participation, it fails to predict decline over time or the number of acts per minute in the last 10 minutes.

5.2.2. Predictions of Model 2, equal and unequal standards of contributive fairness

Model 2 is based on Equations (2.1) and (2.2) (Section 3.2). This model assumes that the amount each actor talks on any given



Group average in the last 10 minutes = 14.9.
Percent of acts by top actor in the last 10 minutes = 21%

Fig. 2. Predicted number of acts per minute by actor, Model 2 contributive standard only, Case 1, equality (all actors aim for 25%).

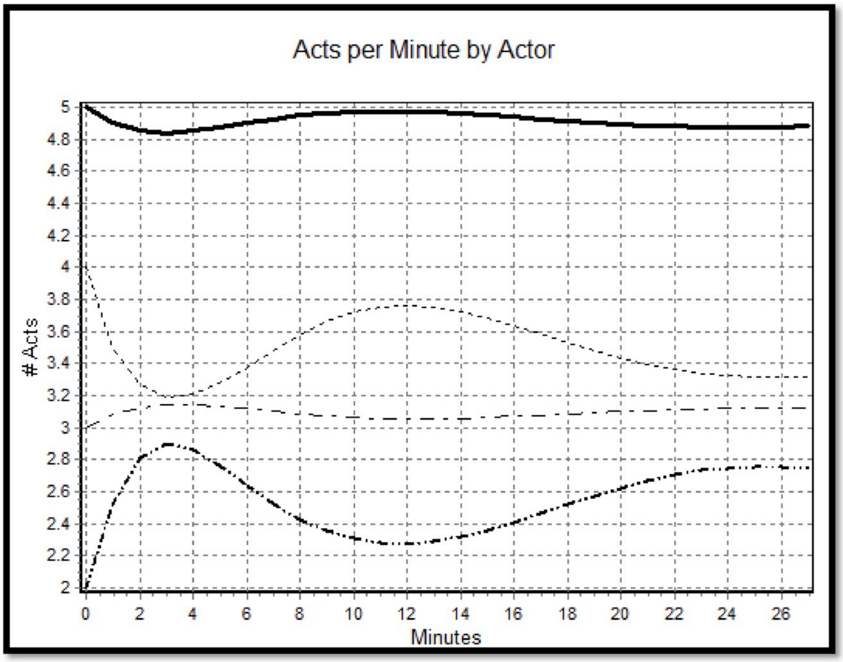
minute is based on the amount he/she talked the minute before but modified now by Φ, the parameter representing a contributive standard. As stated above in Section 5.1, values for unequal Φ are as follows: #1 aims for 35%, #2 aims for 25%, #3 aims for 22%, and #4 aims for 18%. Equations (2.1) and (2.2) with these parameters now are:

Model 2, Case 1, Equality	Model 2, Case 2, Inequality
$x1,t = x_{1,t-1}\,(.25 / a_{1,t-1})$ (6.1)	$x_{1,t} = x_{1,t-1}(.35 / a_{1,t-1})$ (6.5)
$x_{2,t} = x_{2,t-1}(.25 / a_{2,t-1})$ (6.2)	$x_{2,t} = x_{2,t-1}\,(.25 / a_{2,t-1})$ (6.6)
$x_{3,t} = x_{3,t-1}\,(.25 / a_{3,t-1})$ (6.3)	$x_{3,t} = x_{3,t-1}\,(.22 / a_{3,t-1})$ (6.7)
$x_{4,t} = x_{4,t-1}(.25 / a_{4,t-1})$ (6.4)	$x_{4,t} = x_{4,t-1}\,(.18 / a_{4,t-1})$ (6.8)

The results of Model 2 assuming the standard is equality is shown in Fig. 2, while assuming it is unequal is shown in Fig. 3.

These figures show that with both cases of Model 2, the amount of talking fluctuates, neither declining over time nor reaching a stable state by minute 27. Not only does the outcome fail to decline over time, the predicted values of the summary statistics in the last 10 minutes are not similar to the real data. In Fig. 2, contributive standard of equality, the group average at the end is 14.45 (close to the average at the beginning) and the percent contributed by the top actor is 24%, close to the ideal of equality but not to the real data in any of the conditions. In Fig. 3, contributive standard of inequality, the group total oscillates but ends very close to the rate at the beginning, and in this case the percent for the top actor, 35%, replicates the rate for the Equal groups, but not for the Unequal groups.

The assumptions behind Model 2 are those of a “balance”, or “control” theory, which is useful in a variety of sociological applications. In control models, feedback processes operate to maintain an ideal (“balanced”) state in a social system. This kind of maintenance of a stable state is what many theorists mean by “equilibrium”. It is interesting to observe that because it does not predict a



Group average last 10 minutes = 14.06
Percent of acts by top actor in last 10 minutes = 35%

Rank of Actor 1st 2nd 3rd 4th

Fig. 3. Predicted number of acts per minute by actor, Model 2 contributive standard only, Case 2, inequality. (#1 aims for 35%, #2 for 25%, #3 for 22%, #4 for 18%).

change in average talking over time it does not by itself explain the emergence of a stable state in these data. . These data show different type of equilibrium.

5.2.3. Predictions of Model 3. individual differences and deference with No contributive standard

Model 3 uses Eqns. (3.1) to (3.4) (Section 3.3). This model assumes that the amount each actor talks is the same as the amount he/she talked the minute before but modified by parameters representing scarcity of time and a deference order, in which #1 always speaks first, #2 second, etc. The actor with highest volubility ($C_1 = 0.45$) is also highest in the deference order.

$$x_{1,t} = x_{1,t-1} + x_{1,t-1}(-.07(x_{1,t-1}) + .45) \tag{7.1}$$

$$x_{2,t} = x_{2,t-1} + x_{2,t-1}(-.05(x_{1,t}) - .07(x_{2,t-1}) + .35) \tag{7.2}$$

$$x_{3,t} = x_{3,t-1} + x_{3,t-1}(-.05(x_{1,t} + x_{2,t}) - .07(x_{3,t-1}) + .25) \tag{7.3}$$

$$x_{4,t} = x_{4,t-1} + x_{4,t-1}(-.05(x_{1,t} + x_{2,t} + x_{3,t}) - .07(x_{4,t-1}) + .15) \tag{7.4}$$

Predictions made by model 3 are shown in Fig. 4.

As shown in Fig. 4, Model 3 produces a decline in number of acts from minute 1 to minute 27 as desired but fails in other respects: the least talkative members have dropped out, and the 89% of acts achieved by the top actor is much larger than the desired 38%. (The outcome predicted by Model 3 is no longer a group discussion; it is mostly a speech by #1 and is headed for an equilibrium in which #1 does all the talking.)

5.2.4. Predictions of Model 4: all processes combined

None of the models thus examined meet all the criteria listed at the end of Section 4, nor do their results support the hypotheses. Models 1 and 2 can produce inequality but do not show a decline over time, while Model 3 shows inequality and a decline from minute 1 to minute 27 but the least talkative members have dropped out, and the percent of acts produced by the top actor is much too high.

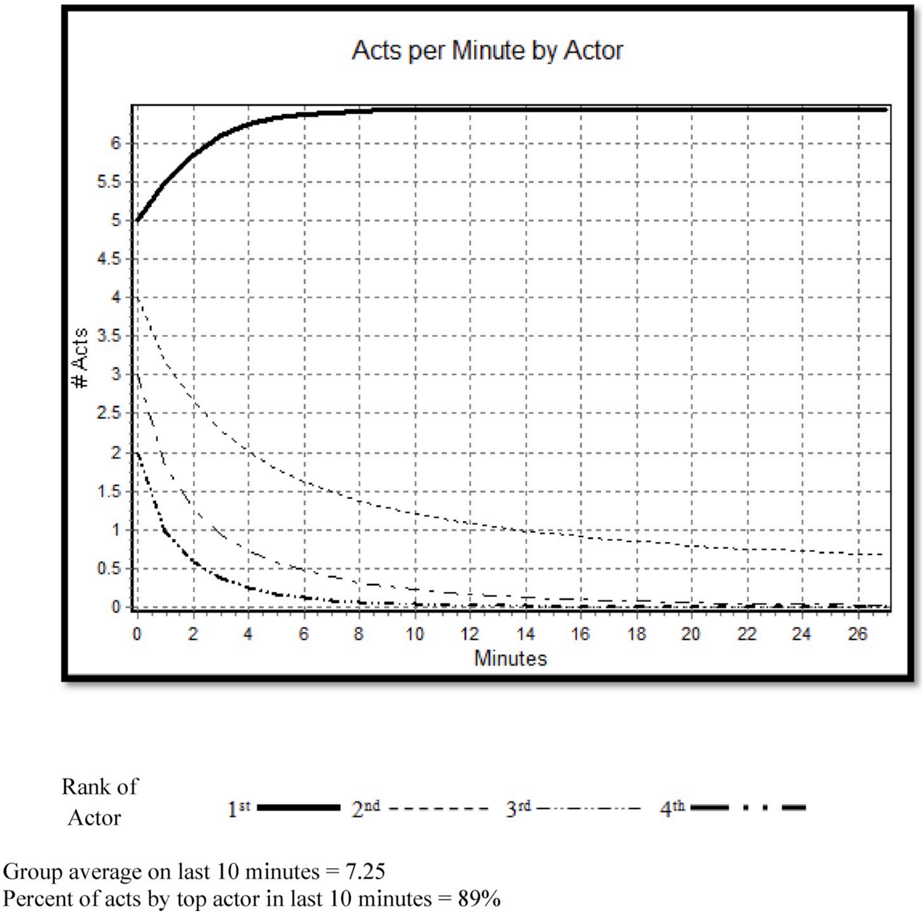


Fig. 4. Predicted number of acts per minute by actor, Model 3 individual differences and deference with no contributive standard.

What will happen if the assumptions of Models 1, 2 and 3 are combined into Model 4, as shown in equations (4.1) - (4.4) (in Section 3.4 above)?

To combine the processes in Models 1 to 3, we need to make some additional assumptions about sources of inequality. First, is the most voluble person also at the top of the deference order? And second, if the most voluble is not first in the deference order does the most voluble think he/she should contribute more than any other, or does the first in the deference order think he/she should talk more than others? In the next three figures, results are presented of three variations of these assumptions.

For the Equal groups it is supposed that the most voluble actor is also at the top in the deference order (i.e. that differences in volubility lead to differences in performance expectations and hence deference) and that there is a contributive standard of equality, as justified above in Section 2.3. In this simulation $\Phi = 25\%$ for all actors.

Model 4, Case 1: most voluble is also first in deference order, contributive standard is equality

$$x_{1,t} = x_{1,t-1}(.25 / a_{1,t-1}) + x_{1,t-1}(-.07(x_{1,t-1}) + .45) \tag{8.1}$$

$$x_{2,t} = x_{2,t-1}(.25 / a_{2,t-1}) + x_{2,t-1}(-.05(x_{1,t}) - .07(x_{2,t-1}) + .35) \tag{8.2}$$

$$x_{3,t} = x_{3,t-1}(.25 / a_{3,t-1}) + x_{3,t-1}(-.05(x_{1,t} + x_{2,t}) - .07(x_{3,t-1}) + .25) \tag{8.3}$$

$$x_{4,t} = x_{4,t-1}(.25 / a_{4,t-1}) + x_{4,t-1}(-.05(x_{1,t} + x_{2,t} + x_{3,t}) - .07(x_{4,t-1}) + .15) \tag{8.4}$$

The results are shown in Fig. 5.

Fig. 5 looks very much like the real groups in the Equal condition: the process declines exactly as in the real data including both the end total and the approximate time when the process stabilizes (about minutes 10–15), each actor develops a stable rate of talking which is different from the others, and no one drops out. For Model 4 Case 1 the group average is 8.76 on minutes 18–27 and the most talkative actor produces 33% of acts. Both values are slightly lower than the real groups but within the 95% confidence interval for Equal groups as shown in Section 4.6. Thus, for both the decline in number of acts and the summary statistics, Model 4 Case 1 replicates

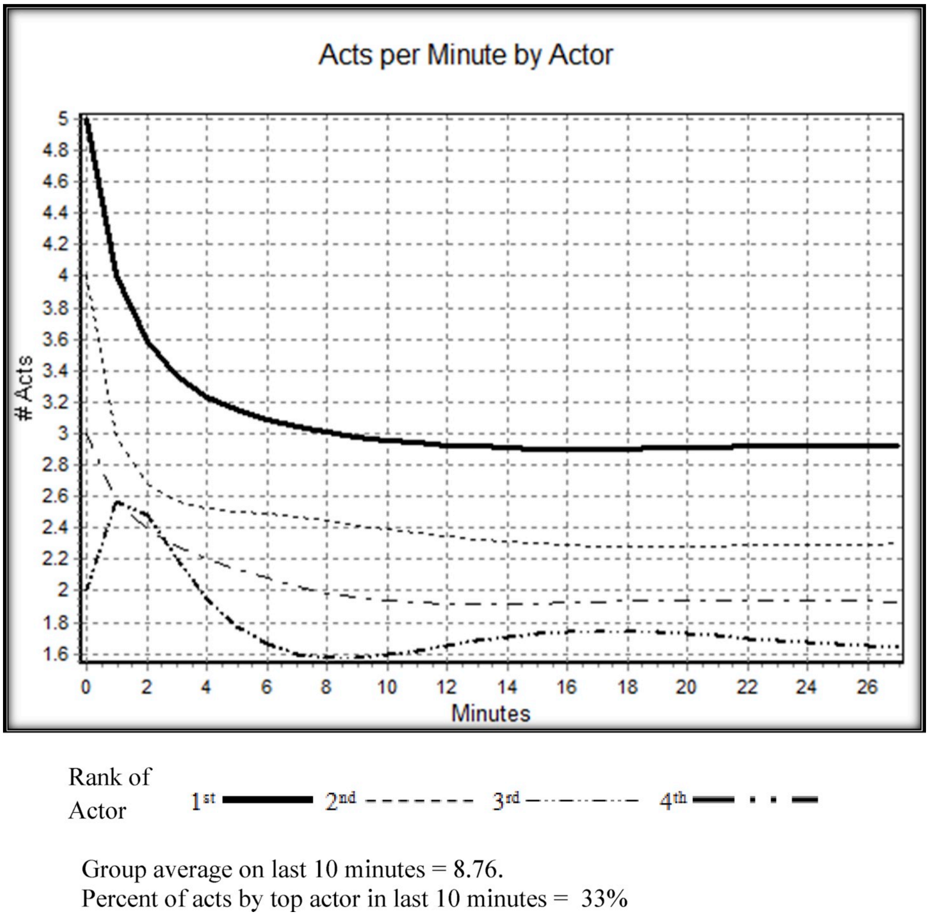


Fig. 5. Acts per minute by actor, Model 4, Case 1, deference and contributive standard of equality, most voluble is also highest in the deference order.

the data from the Equal condition groups.

Although there is justification for assuming a contributive standard of equality for groups in which there is no external status difference (see Section 2.3 above) we might ask why not use a standard of inequality favoring the most voluble, given that we are assuming the existence of a deference order favoring the most voluble. The result of this simulation (the picture is not shown here) is that the output also decreases over time, reaching equilibrium at about minutes 10 to 15, and with a group average of 9.03 acts per minute for minutes 18 to 27. However, the average percent of acts by the most talkative in minutes 18 to 27 is 43%, not within the confidence interval for Equal groups. It appears that a model that best describes the data assumes a restraint on the talkativeness of the naturally voluble (through the contributive standard of equality) as well as an enabling of it (through the deference order).

Although Model 4, Case 1 works well as a simulation of the Equal groups, it does not do as well for the Unequal groups, which are more differentiated. In particular, we are interested in possible explanations of how it happens that in the two Unequal conditions some groups accept the external status information and others do not. For those groups that accept the status information, the highest in external status emerges as the most talkative while in the other groups it is a different group member. We begin with an assumption that the difference may be related to differences in contributive standard and in volubility. To model this, we will assume that the standard of contributive fairness is unequal, using the parameter values for Φ as estimated in Section 5.1. This supposes that the effect of an external status difference is to activate a both a deference order and a contributive standard of inequality. Since status is assigned from outside the groups, we cannot assume that the highest status person will also be the most voluble. For simplicity we assume the most voluble is second in deference order. Should the higher status person, who is first in the deference order, also be the one that the standard of contributive fairness says should talk more than any other? Or should the most voluble, who is second in the deference order, be the one who expects to talk more than any other? We examine two versions of this model, one that makes each of these assumptions. As the next figures will show, the first, Model 4, Case 2 appears to apply to the Unequal Consistent groups and the second, Model 4, Case 3, to the Unequal Inconsistent groups.

Equations for Model 4, Case 2: most voluble is second in deference order: contributive standard unequal and same as deference order are:

$$x_{1,t} = x_{1,t-1}(.35 / a_{1,t-1}) + x_{1,t-1}(-.07(x_{1,t-1}) + .35) \tag{9.1}$$

$$x_{2,t} = x_{2,t-1}(.25 / a_{2,t-1}) + x_{2,t-1}(-.05(x_{1,t}) - .07(x_{2,t-1}) + .45) \tag{9.2}$$

$$x_{3,t} = x_{3,t-1}\ (.22 / a_{3,t-1}) + x_{3,t-1}(-.05(x_{1,t} + x_{2,t}) - .07(x_{3,t-1}) + .25) \tag{9.3}$$

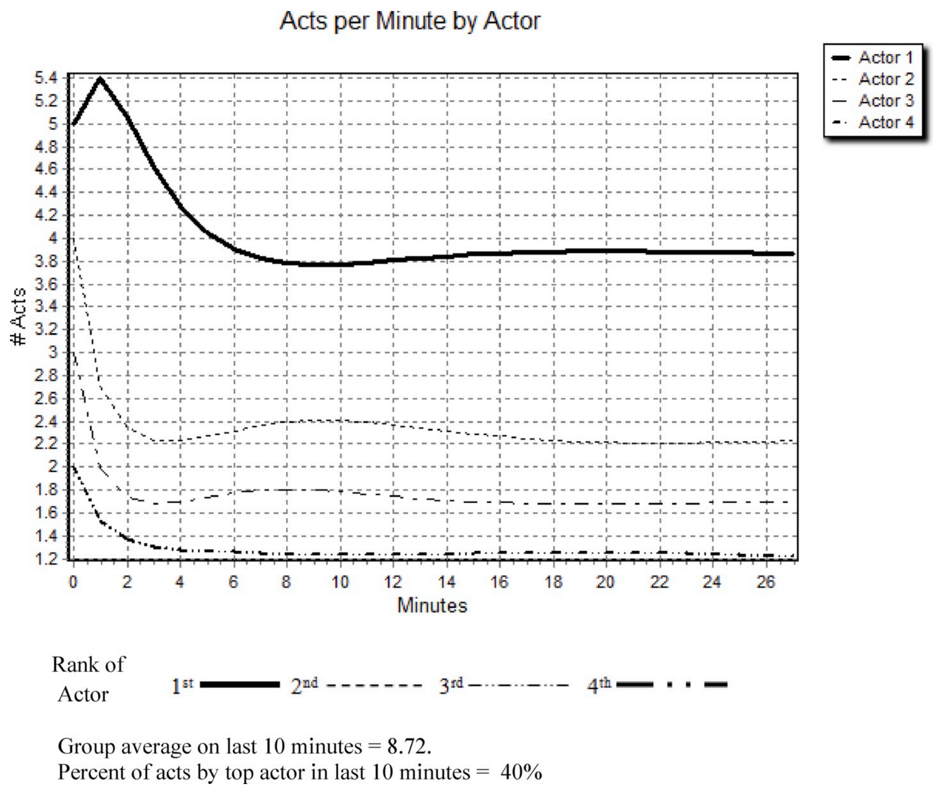


Fig. 6. Model 4, Case 2, deference and unequal contributive standard, most voluble is second in the deference order and the contributive standard advantages the highest in the deference order.

$$x_{4,t} = x_{4,t-1}\,(.18/a_{4,t-1}) + x_{4,t-1}(-.05(x_{1,t}+x_{2,t}+x_{3,t}) - .07(x_{4,t-1}) + .15) \quad (9.4)$$

The results of this simulation are shown in Fig. 6.

This model works well for the Unequal Consistent groups. There is the predicted decrease in group acts over time and no one drops out. For Model 4 Case 2 the group average in minutes 18–27 is 8.72, within the range for the real data for all group conditions. The percent of acts from the most talkative is 40%, within the range for the real data from the Unequal, Consistent condition.

The Unequal Inconsistent groups present a paradox; they seem to respond to the external status difference in that they are more differentiated than the Equal groups but the most talkative is not a person identified by the experimenter as highest status. We resolve this paradox by supposing that these groups accept that a person with higher external status should be first in the deference order, and also that there is a standard of contributive fairness of inequality. However, for Model 4, Case 3, it is assumed that the standard of fairness advantages the most voluble rather than the highest in external status.

Equations for Model 4, Case 3.

$$x_{1,t} = x_{1,t-1}(.25/a_{1,t-1}) + x_{1,t-1}(-.07(x_{1,t-1}) + .35) \quad (10.1)$$

$$x_{2,t} = x_{2,t-1}(.35/a_{2,t-1}) + x_{2,t-1}(-.05(x_{1,t}) - .07(x_{2,t-1}) + .45) \quad (10.2)$$

$$x_{3,t} = x_{3,t-1}\,(.22/a_{3,t-1}) + x_{3,t-1}(-.05(x_{1,t}+x_{2,t}) - .07(x_{3,t-1}) + .25) \quad (10.3)$$

$$x_{4,t} = x_{4,t-1}\,(.18/a_{4,t-1}) + x_{4,t-1}(-.05(x_{1,t}+x_{2,t}+x_{3,t}) - .07(x_{4,t-1}) + .15) \quad (10.4)$$

The results of this simulation are shown in Fig. 7.

Model 4, Case 3 resembles the data from the groups in which the person who is observed to talk the most is not the person with the highest status (the Unequal Inconsistent groups); the average group total for the last 10 minutes is 8.98 and the percent contributed by the most talkative actor in the last 10 minutes is 38%. The figure shows how the most talkative moves from second place on minute 1 to

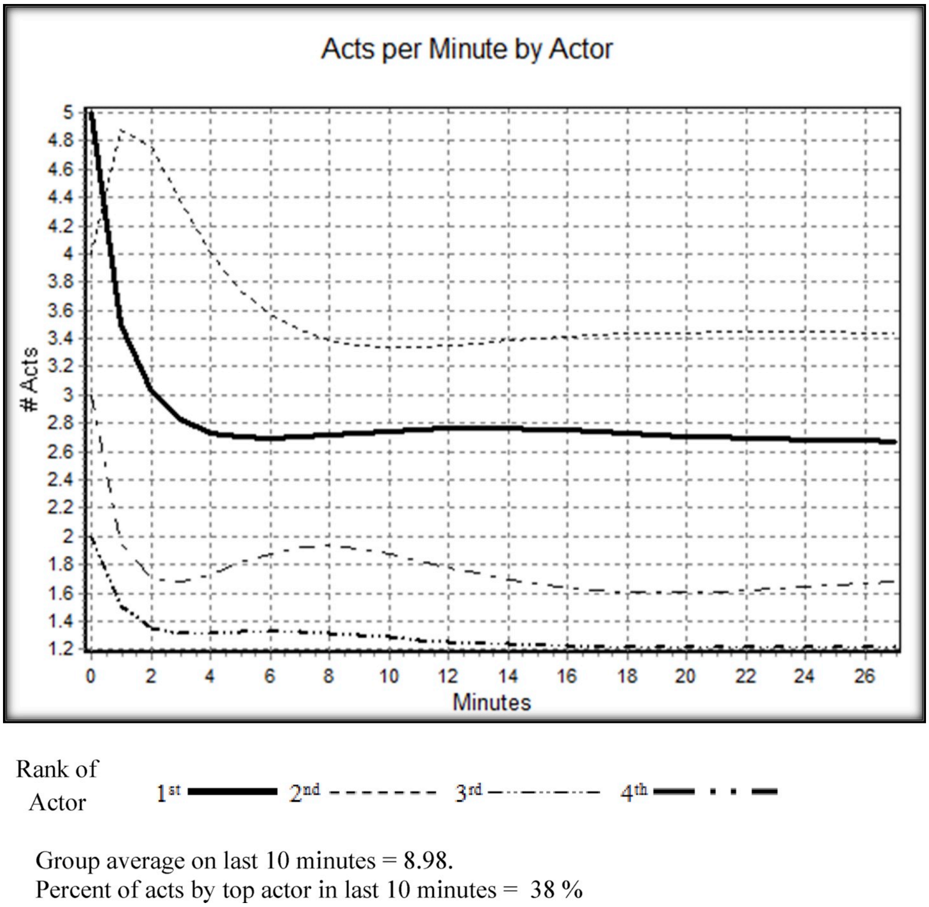


Fig. 7. Model 4, Case 3, Deference and unequal contributive standard, most voluble actor is second in the deference order and unequal contributive standard advantages the most voluble.

most talkative on later minutes.

6. Discussion and some thought experiments

The research question of whether a set of parameters can be found that when used in the equations can predict some real data is answered, yes; Model 4, which combines assumptions from all the other models, predicts both a decline in amount of talking over time and continuous participation as well as values for summary variables that are close to real data. In Model 4 there are three sources of inequality in discussion groups: individual differences in volubility; deference order of speaking; and contributive standard. Each individual begins with a different personal trait of volubility, group members agree on the normative standards of deference order and each has an image of his/her appropriate standard of contributive fairness. Model 4 Case 1 predicts amount of differentiation in groups with no external status differences, Model 4 Case 2 and Case 3 predict differentiation in groups with a pre-assigned external status but different assumptions about standard of contributive fairness.

6.1. *Explaining decrease in amount of talking*

The decline in amount of talking over time in Model 4 does not require positing any change in an underlying state or process such as movement from one phase of group problem solving to another. Mathematically the amount of talking in the last 10 minutes is an equilibrium state toward which the groups are heading from the beginning; it is inherent in the parameters with which it has been assumed the model starts. The model is an example of a nonlinear dynamical system with parameters in a range in which the output reaches a "fixed point"; in other words, an equilibrium that will not change unless interrupted from outside the system. It also means that the number of acts by each actor in minute 1 is not what determines the final outcome. If all the same parameters as Model 4, Case 1 are used except with the assumption that all the actors begin with a low number of acts (for example, 2 each in minute 1), the result by the last 10 minutes is exactly the same as found for Model 4 Case 1 (Fig. 5 above). However, if the number of acts on minute 1 has been low the rate of talking will *increase* rather than decreasing over time. (We might picture a group in which members spend the first minute with low rates of talking, perhaps introducing themselves, or giving a short statement about the task before beginning open discussion. Once the discussion is underway, the processes of deference and contributive standard together with individual volubility increase talking until the fixed point is reached.) This is in contrast with other typical models of group interaction that suppose that a

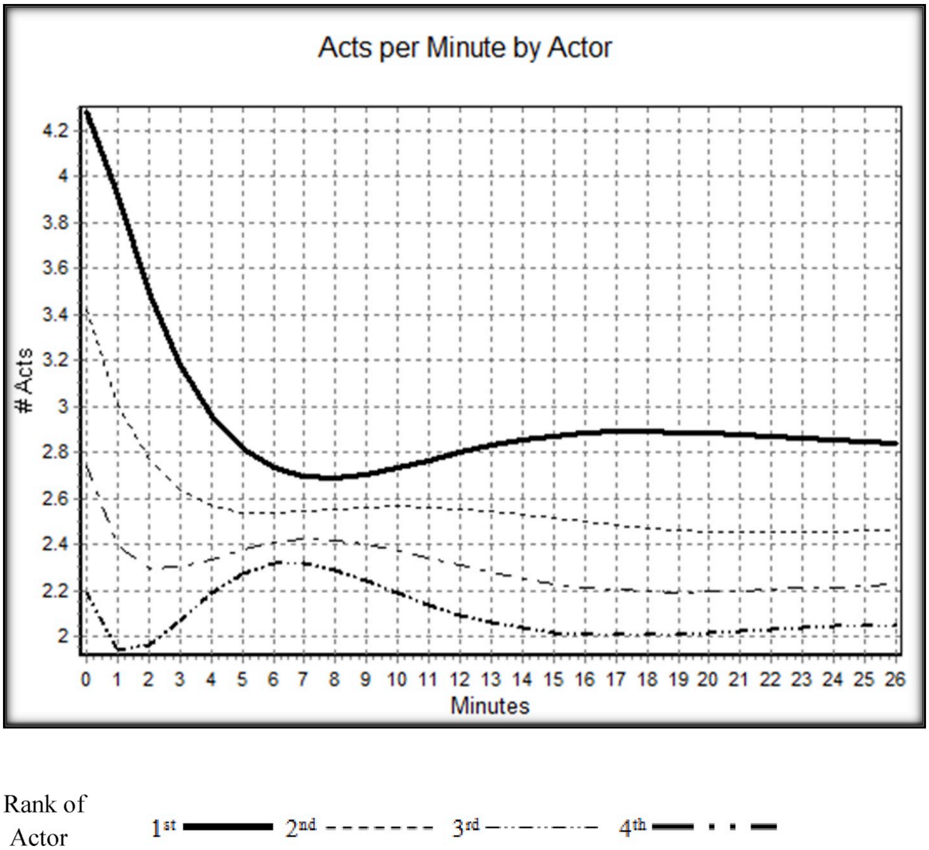


Fig. 8. Model 4 Case 1 but with all parameters set to equality.

change in observable behavior must represent a change in some underlying state (members have become less voluble, the group has moved into a phase of problem solving requiring less talking, a deference order has emerged or changed).

6.2. Relationships among sources of inequality

This model allows us to ask how several processes can operate at once. In fact, it suggests that we can only understand each separate process by seeing how it combines with others. In Model 4 some of the processes seem to be working at cross purposes; as one process makes the dominant talker talk more, others reduce this rate of talking. However, it is exactly the tension between these processes that creates equilibrium, keeping all group members participating and resulting in a stable rate of talking for each member.

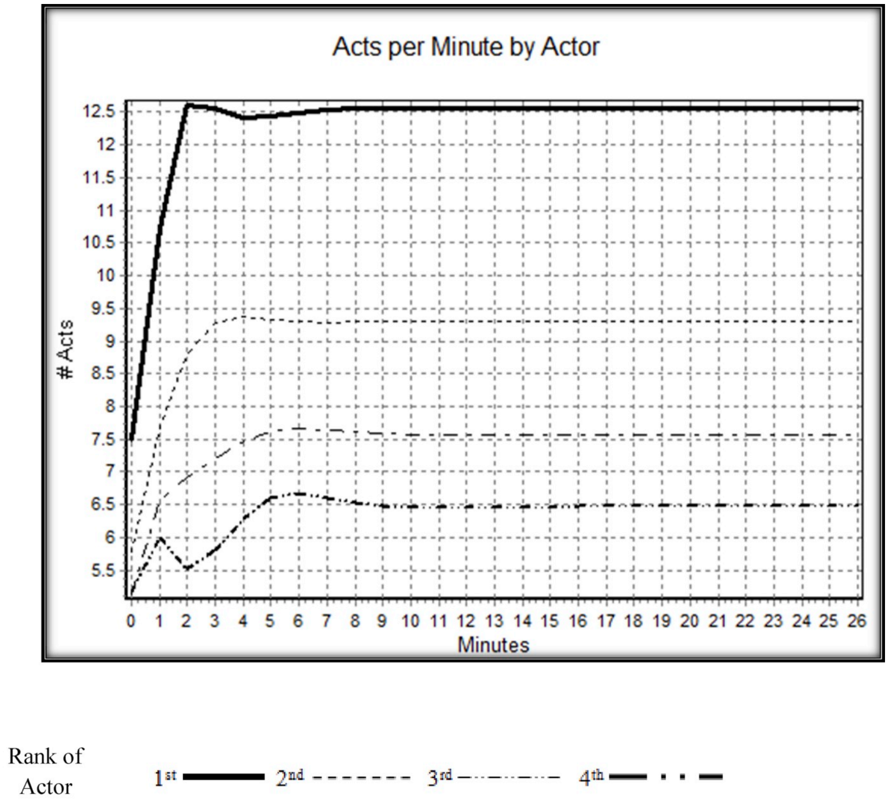
One of the assumptions behind Model 4, Case 1, which best fits the data from the Equal experimental condition, is that in the absence of any external status differences, deference is allocated on the basis of volubility. Model 4, Cases 2 and 3 suppose that external status differences activate both a deference process in which a group member with higher external status is granted the legitimate right to speak first and a standard of contributive fairness of inequality. However, in Model 4 Case 2 it is the highest status person who is also identified by the contributive standard as the one who should talk the most while in Model 4 Case 3 the contributive standard identifies the most voluble rather than the highest status as deserving to talk more than others. This offers a picture of how it can happen that information about external status can increase differentiation in a group while at the same time the most dominant member is not the one with highest external status.

6.3. Some thought experiments

An advantage of a mathematical model is that it allows for answering hypothetical questions about what would happen under different conditions. In this section, several "thought experiments" explore such questions.

6.3.1. How much of inequality is due to deference of speaking order alone?

Of the processes that contribute to inequality of talking; how much of the inequality is due to being the first to talk in a situation where opportunities to talk are scarce? Fig. 8 answers this. Here all the other parameters representing sources of inequality are set to be



Group average on last 10 minutes = 35.90
Percent of acts by top actor in last 10 minutes = 35%

Fig. 9. Model 4 Case 1 with all four actors expecting to talk 50% of the time.

equal; members are equally voluble (the parameter C is the same for all group members), the contributive standard is assumed to be equality (Φ = 25% for all actors) and this model assumes all members start with the same number of acts on the first minute (4 acts for each). As Fig. 8 shows, the proportion of acts by the most talkative is only slightly reduced compared to Fig. 5: about 30% of acts come from the most talkative. Speaking first provides an advantage even when all other processes point to equality.

6.3.2. What if several group members each expect to talk more than the others?

Another hypothetical question comes from the assumptions in Models 2 and 4 that the contributive standards of all group members add to 100%. Any one who has ever sat on a committee knows that there may be more than one person in a discussion group who thinks he/she should talk more than others. Fig. 9 shows what happens if one makes all the same assumptions as for Model 4, Case 1 except for assuming all four actors each think they should talk 50% of the time.

The result here is that the system still reaches a fixed point, no actor completely drops out, and the top actor speaks a little more than a third of the time on minutes 18–27. However, there is a very high rate of talking (group total is 35.90) which suggests that all members are talking at once.

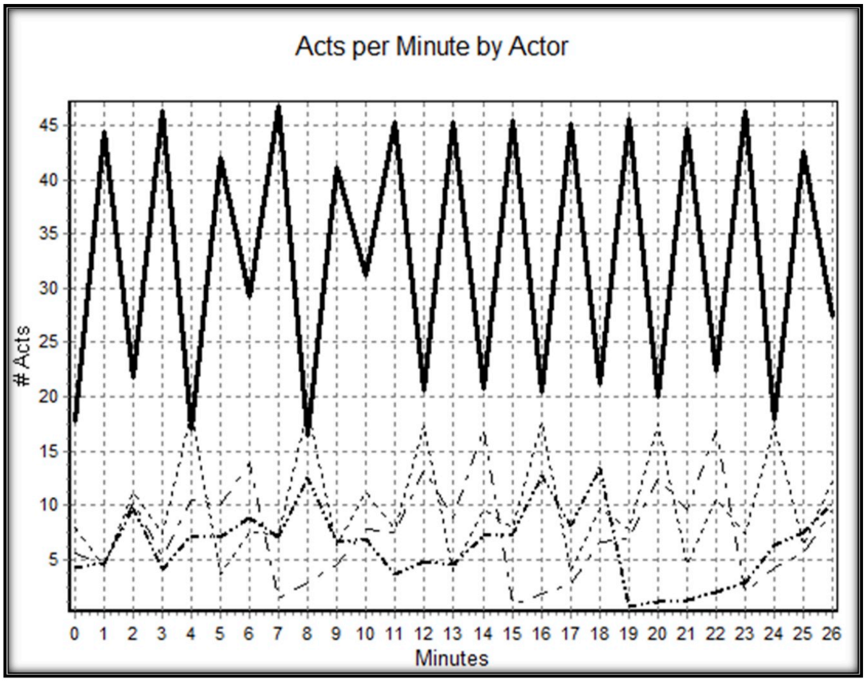
6.3.3. Will applications of this model always reach a fixed point?

These thought experiments have shown examples in which the basic shape of the process, which stabilizes at a fixed point, endures through several variations in parameters. A final hypothetical question or "thought experiment" is whether, like other nonlinear models based on variations of the logistic curve, Model 4 can become chaotic under some parameter values. Fig. 10 shows the answer.

The answer is yes; Fig. 10 shows that by substantially increasing the values of C, the parameter that represents individual volubility, the process can be made to become chaotic. However, it is unlikely that any actual groups will get to this point, as the average amount of total group talking is more than 50 acts per minute.

7. Summary and conclusions

This paper has introduced a nonlinear dynamical system model to add to the toolkit of mathematics used to analyze interpersonal interaction in group settings. It begins with a familiar question in sociology of group processes: why some people talk more than others in a group discussion. The task of group discussion requires taking turns which means that participants must coordinate actions in a



Rank of Actor 1st 2nd 3rd 4th

Group average on last 10 minutes =56.01
Percent of acts by top actor in last 10 minutes = 58%

Fig. 10. Model 4 Case 1 except that the values for parameter C are: #1 = 3.20, #2 = 2.30, #3 = 2.20 and #4 = 2.10. The system has become chaotic.

situation of the scarce resource of time. Thus, a dynamical system model originally derived from biology that describes individual organisms in a situation where they must share scarce resources is theoretically appropriate. Within that structure, models are proposed that incorporate processes that have through the years been hypothesized by sociologists to affect participation in group discussion. These include inherent individual differences, deference and status generalization. One previously less examined process, effect of a standard of contributive fairness, is introduced and shown to be necessary to produce adequate simulations of a data set. Some of these underlying processes may seem to contradict each other, but it is the conflicting pressures that produce and maintain a stable but differentiated pattern of interaction. The most successful models picture status generalization operating through two processes: deference on the immediate minute (higher status persons speak first) and through a standard of contributive fairness (higher status persons are expected to talk more than others over the course of the discussion).

Acknowledgments

I am grateful to John Skvoretz, Murray Webster and Joseph Whitmeyer for sharing the data used here. They are not responsible for any of the analysis or interpretations in this article. Their data collection was supported by NSF grants SBR-911514 and SBR-911527.

An earlier version of some of this paper was presented at the American Sociological Association, Montreal, Quebec, Canada, August 2017.

References

Bales, R.F., 1950. Interaction Process Analysis: A Method for the Study of Small Groups. Addison-Wesley, Cambridge, MA.
Bales, R.F., 1952. The equilibrium problem in small groups. In: Parsons, T., Bales, R.F., Shils, E.A. (Eds.), Working Papers in the Theory of Action. The Free Press, Glencoe, IL, pp. 111–161.
Bales, R.F., Slater, P.E., 1955. Role differentiation in small decision-making groups. In: Parsons, T., Bales, R.F. (Eds.), Family, Socialization and Interaction Process. The Free Press, Glencoe, IL, pp. 259–306.
Bales, R.F., Strodtbeck, F.L., Mills, T.M., Roseborough, M.E., 1951. Channels of communication in small groups. Am. Sociol. Rev. 16, 461–468.
Berger, J., Cohen, B.P., Zelditch Jr., M., 1972. Status characteristics and social interaction. Am. Sociol. Rev. 37, 241–255.
Berger, J., Connor, T.L., Fisek, M.H., 1974. Expectation States Theory: A Theoretical Research Program. Winthrop, Cambridge, MA.
Berger, J., Webster, M., Ridgeway, C.L., Rosenholtz, S.J., 1986. Status cues, expectations, and behaviors. In: Lawler, E.J. (Ed.), Advances in Group Processes, vol. 3. JAI Press, NY, pp. 1–22.
Borgatta, E.F., Bales, R.F., 1953. Interaction of individuals in reconstituted groups. Sociometry 16, 302–320.
Burke, P., 1974. Participation and leadership in small groups. Am. Sociol. Rev. 39, 832–843.
Conner, T.L., 1977. Performance expectations and the initiation of problem solving attempts. J. Math. Sociol. 5, 187–198.
Fisek, M.H., Berger, J., Norman, R.Z., 2005. Status cues and the formation of expectations. Soc. Sci. Res. 34, 80–102.
Horvath, W.J., 1965. A mathematical model of participation in small groups. Behav. Sci. 10, 164–166.
Kadane, J.B., Lewis, G.H., 1969. The distribution of participation in group discussions: an empirical and theoretical reappraisal. Am. Sociol. Rev. 34, 710–723.
Leik, R.K., Meeker, B.F., 1995. Computer simulation for exploring theories: models of interpersonal cooperation and competition. Sociol. Perspect. 38, 463–482.
Lotka, A.J., 1932. The growth of mixed populations: two species competing for the common food supply. J. Wash. Acad. Sci. 21, 461–469.
Meeker, B.F., 2002. Some philosophy of science problems in the development of complex computer simulations. In: Szmatka, J., Lovaglia, M.J., Wysienska, K. (Eds.), The Growth of Social Knowledge: Theory, Simulation, and Empirical Research in Group Processes. Praeger, Westport, CT, pp. 183–202.
Meeker, B.F., Leik, R.K., 1997. Computer simulation: an evolving component of theoretical research programs. In: Szmatka, J., Skvoretz, J.V., Berger, J. (Eds.), Status, Network and Structure: Theory Construction and Theory Development. Stanford Univ. Press, Palo Alto, CA, pp. 47–70.
Reynolds, P.D., 1971. Comment on "The distribution of participation in group discussions as related to group size". Am. Sociol. Rev. 36, 704–706.
Ridgeway, C.L., 2014. Why status matters for inequality. Am. Sociol. Rev. 79, 1–16.
Ridgeway, C.L., Walker, H., 1955. Status structures. In: Cook, K.S., Fine, G.A., House, J.S. (Eds.), Sociological Perspectives on Social Psychology. Allyn & Bacon, Needham Heights, MA, pp. 281–310.
Robinson, D.T., 2007. Control theories in sociology. Annu. Rev. Sociol. 33, 157–174.
Skvoretz, J., Fararo, T.J., 1996. Status and participation in task groups. Am. J. Sociol. 10, 1366–1414.
Skvoretz, J., Fararo, T.J., 2016. Status and interaction: a stochastic model for the measurement of macro-status value and the determination of micro-status ranking in task group interaction. Soc. Sociol. Res. Dyn. World 2, 1–16.
Skvoretz, J., Webster, M., Whitmeyer, J., 1999. Status orders in task discussion groups. Adv. Group Process. 16, 199–218.
Strogatz, S.H., 2015. Nonlinear Dynamics and Chaos: with Applications to Physics, Biology, Chemistry, and Engineering, second ed. CRC Press, NY.
Volterra, V., 1931. Leçon sur la theorie mathematique de la lutte pour la vie. Gauthier-Villars, Paris.